07-11-2011/mkn

# Kandidatkurser for Foråret 2012

## Matematik

Algebra gruppen

| Dansk titel | Engelsk titel | Underviser |
|---|---|---|
| Matematiske aspekter af kryptologi | Mathematical aspects of cryptology | Jørgen Brandt |
| Lie algebraer | Lie Algebras | Henning Haahr Andersen |
| Algebra og Polyhedrael Geomeri | Algebra and Polyhedral Geometry | Anders Nedergaard Jensen |

Analyse gruppen

| Dansk titel | Engelsk titel | Underviser |
|---|---|---|
| Videregående analyse | Advanced analysis | Jacob Schach Møller |
| Analyse på mangfoldigheder | Analysis on manifolds | Bent Ørsted |
| Partielle differentialligninger | Partial differential equations | Erik Skibsted |
| Videregående kompleks funktionsteori | Advanced complex function theory | Søren Fournais |
| Funktionalligninger II | Functional equations II | Henrik Stetkær |
| Videregående talteori | Further Number Theory | Simon Kristensen og Alexei Venkov |
| Matematisk projektarbejde | Project work in mathematics | Forskellige undervisere |

Topologi gruppen

| Dansk titel | Engelsk titel | Underviser |
|---|---|---|
| Kohomologi og homotopiteori | Cohomology and homotopy theory | Andrew du Plessis (Jørgen Tornehave er konsulent på kurset) |
| Riemannsk geometri | Riemannian Geometry | Andrew Swann (Jørgen Tornehave er konsulent på kurset) |

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# UNDERVISNINGSBESKRIVELSE

## Mathematical aspects of cryptology (Q3+Q4)

### Objectives of the course

The course covers mathematical aspects necesssary for the design and evaluation of secure public key crypto systems.

### Compulsory programme

None

### Course contents

Public key crypto systems like RSA and DSA are based on algebraic structures, in particular groups, and prime numbers play a fundamental role. The mathematical problems that make the systems possible are factorizing of integers and the discreet logarithm problem.

The course covers the prime number theory necessary for the construction of practical systems, e.g. prime number tests and algorithms for efficient calculation with very large integers. Factoring algorithms from test division through Pollard's probabilistic methods, the n-1 algorithm and Lenstra's method, to the strongest algorithms that in different non-trivial ways reduce the problem to linear algebra. Understanding these algorithms is essential for the construction of secure systems.

Methods for solving the discrete logarithm problem are examined, e.g. Shank's baby step giant step method, Pollard's rho algorithm, Pohlig-Hellman and index calculus. Not all algorithms are generic, some depend on the structure of the group order (flatness) others on the concrete representation of the group. This is important in connection to elliptic curves. Here the best algorithm does not work. This translates directly to the same security for much smaller group orders compared to finite fields. Knowledge of these attacks is a prerequisite for the construction of secure systems.

Elliptic curves are treated to the extent necessary for understanding how systems can be implemented there.

### Prerequisites

Algebra.

### Name of lecturer

Jørgen Brandt

## Type of course / teaching methods

4 hours of lectures per week.

## Literature

Crandall & Pomerance, *Prime Numbers, 2.nd ed.*, Springer, 2005

## Language of instruction

English

## Course homepage

You will be able to find the course homepage at http://www.imf.au.dk before the course starts.

## Level of course

Graduate course

## Semester/quarter

3rd + 4th quarter (Spring 2012)

## Hours per week

4

## Capacity limits

## Assessment methods

The course will be evaluated using the Danish 7-scale with an internal examiner.

The evaluation is divided into two test points.

The first test point is a written assignment.

The second test point is an oral examination lasting 20 minutes after 25 minutes preparation and with the use of all usual means of aid.

In determining the grade the first test point weighs 1/3 and the oral examination weighs 2/3.

## Credits

10

ECTS

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher

## Provider

Department of Mathematical Sciences.

## Course enrolment

At the self-service https://mit.au.dk

## Special comments on this course

None

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to

- > describe the mathematical principles behind modern public key cryptography
- > evaluate the security of concrete systems
- > compare key results,
- > apply the basic techniques, results and concepts of the course to concrete examples and exercises,
- > to discuss a prescribed topic by applying the course theory to the topic,
- > combine concepts from algebra and number theory.

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# UNDERVISNINGSBESKRIVELSE

## Lie Algebras (Q3+Q4)

### Objectives of the course

To introduce Lie Algebras and study their representations

### Compulsory programme

A participant may only take the final examination if he of she has handed in, and had approved, at least 4 out of 6 set exercises.

### Course contents

This will be an introduction to a large field in modern Mathematics, namely Lie theory. Lie algebras are named after the Norwegian mathematician Sophus Lie (1842-1899). They play a role in several areas of mathematics. Our treatment will be purely algebraic and we will almost exclusively deal with finite dimensional complex Lie algebras.

First we shall give the basic definitions and first properties of Lie algebras. We will discuss many examples and concrete realizations. Then we shall study three important classes: the nilpotent, the solvable and the semisimple Lie algebras. The latter class will be the object for an intensive investigation. The highlight will be a theorem (the main result in "the greatest mathematical paper of all time", see Math. Intel. **11** no. 3, 29-38) which classifies them in terms of their socalled root systems.

Throughout this structure theory we shall emphasize the representations of Lie algebras. The course will end by a closer look at some of the main features of representations of semisimple Lie algebras.

### Prerequisites

Algebra

### Name of lecturer

Henning Haahr Andersen

## Type of course / teaching methods

4 hours of lectures per week including exercises

## Literature

J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Graduate Text in Mathematics, Springer-Verlag
or
R. Carter, *Lie Algebras of Finite and Affine Type*, Cambridge Studies in advanced mathematics.

## Course homepage

You will be able to find the course homepage at
http://www.imf.au.dk before the course starts.

## Language of instruction

English

## Level of course

Graduate course

## Semester/quarter

3rd + 4th quarter (Spring 2012)

## Hours per week

4

## Capacity limits

## Assessment methods

The course will be evaluated with an oral examination lasting about 20 minutes, without preparation. We use the Danish 7-scale and an internal examiner.

## Credits

10 ECTS

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher

## Provider

Department of Mathematical Sciences

## Course enrolment

At the self-service https://mit.au.dk

## Special comments on this course

None

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- reproduce key results and give rigorous and detailed proofs of them,
- compare key results,
- apply the basic techniques, results and concepts of the course to concrete examples and exercises,
- combine concepts from the course with other important topics in algebra.

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## Algebra and Polyhedral Geometry (Q3+Q4)

### Objectives of the course

To introduce algorithmic, combinatorial and polyhedral methods in the study of polynomial ideals.

### Course contents

This is an introduction to computational algebra emphasizing the connection between polynomial ideals and convex polytopes. It builds on the concepts of polynomial rings, ideals and Gröbner bases introduced in Algebra.

Subjects to be discussed in this course include:

Newton polytopes of polynomials, initial ideals, variation of termorders in Buchberger's Algorithm and Gröbner fans.

Applications of Gröbner bases and Buchberger's Algorithm to integer programming and toric ideals.

Polyhedral complexes, regular triangulations of point configurations and secondary fans.

Tropical varieties arising as tropicalisations of classical varieties.

### Prerequisites

### Name of lecturer

Anders Nedergaard Jensen

### Type of course / teaching methods

4 hours of lectures per week including exercises.

### Literature

Diane Maclagan and Rekha Thomas: *Computational Algebra and Combinatorics of Toric Ideals*, Ramanujan Mathematical Society Lecture Notes Series, also available online.

## Course homepage

You will be able to find the course homepage at http://www.imf.au.dk before the course starts.

## Language of instruction

English.

## Level of course

Graduate course

## Semester/quarter

3rd + 4th quarter (Spring 2012).

## Hours per week

4

## Capacity limits

## Assessment methods

The course will be evaluated using the Danish 7-scale with an internal examiner.

The evaluation is divided into two test points.

The first test point is a written assignment.

The second test point is an oral examination lasting about 20 minutes after 25 minutes preparation. and with the use of all usual means of aid.

In determining the grade the first test points weighs 1/3, the oral exam weighs 2/3.

## Credits

10 ECTS

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher

## Provider

Department of Mathematical Sciences.

## Course enrolment

At the self-service https://mit.au.dk

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- reproduce key results and give rigorous and detailed proofs of them,
- compare key results,
- apply the basic techniques, results and concepts of the course to concrete examples and exercises,
- carry out computations of examples in a computer algebra system,

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# UNDERVISNINGSBESKRIVELSE

## Advanced Analysis (Q3+Q4)

### Objectives of the course

The main purpose of the course is to give an introduction to the basic concepts of advanced (functional) analysis by introducing some of the fundamental tools for handling of Hilbert space operators.

### Compulsory programme

Participation in exercise class.

### Course contents

The course will continue the introduction to functional analysis begun in the courses 'Real Analysis' and 'Measure Theory'. Starting with an introduction to the theory of topological vector spaces we will obtain a description of commutative $C^*$-algebras. As an application we will then develop the spectral theory of a bounded normal operators on a Hilbert space; first the continuous function calculus, which follows readily from the theory of commutative $C^*$-algebras, and then, subsequently, the Borel-function calculus, which uses the abstract integration theory from 'Measure Theory'. Additional subjects, related to the above, will be introduced to the extent time permits it.

As the course proceeds, we will identify areas and subjects related to the content of the course and/or text-book which could be used to take the course as a bachelor project for students who wish to do so.

### Prerequisites

The courses *Real Analysis* and *Measure Theory*.

### Name of lecturer

Jacob Schach Møller

### Type of course / teaching methods

4 hours of lectures per week including exercises.

## Literature

Notes.

## Course homepage

You will be able to find the course homepage at http://www.imf.au.dk before the course starts.

## Language of instruction

English

## Level of course

Graduate course

## Semester/quarter

3rd + 4th quarter (Spring 2012)

## Hours per week

4

## Capacity limits

## Assessment methods

The course will be evaluated using the Danish grading scale (the 7-point scale) with an external examiner.

If the course is taken as a bachelor project, then the course will be evaluated via a longer written assignment.

If the course is not taken as a bachelor project, then the evaluation is divided into two test points.

The first test point is a written assignment that must be handed in to the teacher at the beginning of the second quarter.

The second test point is an oral examination lasting about 25 minutes after 30 minutes preparation.

In determining the grade the first test points weighs 1/3 and the oral exam weighs 2/3.

## Credits

10

ECTS

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher

## Provider

Department of Mathematical Sciences.

## Course enrolment

At the self-service https://mit.au.dk

## Special comments on this course

None.

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- › reproduce key results and give rigorous and detailed proofs of them,
- › compare key results,
- › apply the basic techniques, results and concepts of the course to concrete examples and exercises,

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# UNDERVISNINGSBESKRIVELSE

## Analysis on manifolds (Q3+Q4)

### Objectives of the course

This course will be an introduction to analysis on manifolds and the connection between the geometry and solutions to differential equations. In particular, following the classical works of Nirenberg, Seeley, and Bott from the 1960's, we shall introduce pseudodifferential operators, zeta functions, and the heat equation, leading to spectral geometry and the index theorem.

### Course contents

This course will be an introduction to spectral geometry, with an emphasis on finding local and global invariants of compact Riemannian manifolds. These invariants arise from studying the spectrum of Laplace-type operators on the manifolds as well as the corresponding heat and wave equations. This is in the spirit of Kac's famous question: Can you hear the shape of a drum? We shall give the necessary background from Riemannian geometry and pseudodifferential operators, and discuss examples in low dimensions. Spectral geometry has several applications in other parts of mathematics (index theory, characteristic classes, differential geometry) and in physics (string theory, gauge theory). Depending on the interests of the audience, we shall give some applications, for example in connection with the study of determinants of elliptic operators.

### Prerequisites

Basic knowledge of manifolds, Fourier transforms, and Hilbert spaces.

### Name of lecturer

Bent Ørsted

### Type of course / teaching methods

4 hours of lectures per week including exercises.

### Literature

The basic textbook will be no. 3 below, but the other references will be useful as well.

1. M. Beals, C. Fefferman, and R. Grossman, *Strictly pseudoconvex domains in $\mathbb C^n$*, Bull. Amer. Math. Soc. **8** (1983), 125-322.

2. T. P. Branson, and B. Ørsted, *Explicit functional determinants in four dimenensions*, Proc. Amer. Math. Soc. **113** (1991), 669-682.

3. P. B. Gilkey, *Invariance Theory, the Heat Equation, and the Atiyah-Singer Index Theorem*, CRC Press 1995.

4. B. Osgood, R. Phillips, and P. Sarnak, *Extremals of determinants of Laplacians*, Journ. Func. Anal. **80** (1988), 148-211.

5. T. Sakai, *Riemannian Geometry*, Amer. Math. Soc. Transl. **149**, 1996.

6. M. A. Shubin, *Pseudodifferential Operators and Spectral Theory*, Springer 2001, 2nd. ed.

## Language of instruction

English

## Course homepage

You will be able to find the course homepage at
http://www.imf.au.dk
before the course starts.

## Level of course

Graduate course

## Semester/quarter

3rd + 4th quarter (Spring 2012)

## Hours per week

4

## Capacity limits

## Assessment methods

The course will be evaluated using the Danish 7-scale with an internal examiner.

The evaluation is divided into two test points.

The first test point is a presentation in class (about 30 minutes) of some part of the course material.

The second test point is an oral examination lasting about 30 minutes with 30 minutes preparation and with the use of all usual means of aid.

In determining the grade the first test points weighs 1/3, the oral exam weighs 2/3.

## Credits

10

ECTS

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher

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## Provider

Department of Mathematical Sciences.

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## Course enrolment

At the self-service https://mit.au.dk

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## Special comments on this course

The topics will in outline be:

- › A. Pseudodifferential operators [3], [6]
- › B. Elliptic operators and Fredholm operators [3], [6]
- › C. Spectral theory and asymptotics of the heat kernel; review of Riemannian geometry [5], [1], [3]
- › D. Zeta and eta functions associated elliptic operators [3], [6], [4], [2]
- › E. Spectral geometry and some applications [4], [2]

We shall follow [3] fairly closely the first 8-9 weeks, pages 1-74, with some extra background as needed from [5] and [1]; see in particular [5] pages 1-19 and the appendix pages 289-292 for background about manifolds, Riemannian geometry, and vector bundles. Then we give a discussion of the classical approach to the Gauss-Bonnet theorem following [1] pages 197-237, about 2 weeks. For spectral geometry we follow [5] pages 282-288 and references there (also [4] and [3] pages 327-344), and in the last 2 weeks we discuss topics, for example determinants (and eta invariants, or torsion invariants) following [4] and [2]. So in outline we shall cover:

1. The Fourier transform, Schwartz space, Sobolev spaces
2. Pseudodifferential operators on $\mathbb R^n$, symbol classes
3. Operators defined by kernels
4. Pseudodifferential operators defined on manifolds
5. Fredholm operators, indices, elliptic operators
6. Pseudodifferential operators defined on manifolds
7. Fredholm operators, indices, elliptic operators
8. Elliptic complexes, spectral theory
9. Heat kernels and index theorems
10. Local index formulas
11. Riemannian geometry, local invariants
12. The Gauss-Bonnet theorem, classical approach
13. Spectral geometry, simple examples
14. Spectral geometry, recent examples
15. Determinants of Laplace-type operators
16. Extremal propreties of determinants, logarithmic Sobolev inequalities.

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## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- › reproduce key results and give rigorous and detailed proofs of them,
- › compare key results,
- › apply the basic techniques, results and concepts of the course to concrete examples and exercises,
- › combine concepts from algebra, analysis and topology, and
- › show, how the course generalizes classical results.

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## Partial differential equations (Q3+Q4)

### Objectives of the course

Analysis of partial differential equations.

### Course contents

We study basic issues of partial differential equations, for example the well-posedness of the so-called Cauchy problem for partial differential equations, which may be viewed as a generalization of the well-known existence and uniqueness properties for the initial value problem of ordinary differential equations. We shall also discuss solutions of concrete equations with other types of initial- or boundary value conditions imposed. Our examples include the Laplace-, the Schrödinger, the wave- and the heat equation.

We shall become acquainted with the following tools or methods: Real analytic functions, the Fourier transform, Sobolev spaces, fundamental solutions, energy- and Hilbert space methods, maximum principles and (maybe) elliptic regularity.

The course may serve as a background for further study of partial differential equations and/or functional analysis.

### Prerequisites

Real analysis, Complex function theory, Differential equations.

### Name of lecturer

Erik Skibsted

### Type of course / teaching methods

4 hours of lectures per week including exercises.

### Literature

L. C. Evans: *Partial differential equations*, GSM American Mathematical Society, 2000, 1. edition.

## Language of instruction

English

## Course homepage

You will be able to find the course homepage at http://www.imf.au.dk/en before the course starts.

## Level of course

Graduate course

## Semester/quarter

3rd + 4th quarter (Spring 2012)

## Hours per week

4

## Capacity limits

## Assessment methods

The course will be evaluated using the Danish 7-scale with an internal examiner.

The evaluation is divided into two test points.

The first test point is a presentation on a given subject.

The second test point is an oral examination lasting about 20 minutes after 25 minutes preparation and with the use of all usual means of aid.

In determining the grade the first test points weighs 1/3, the oral exam weighs 2/3.

## Credits

10 ECTS

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher.

## Provider

Department of Mathematical Sciences (IMF).

## Course enrolment

At the self-service https://mit.au.dk/index.cfm?sp=en

## Special comments on this course

None

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- reproduce key results and give rigorous and detailed proofs of them,
- compare key results,
- apply the basic techniques, results and concepts of the course to concrete examples and exercises,
- combine concepts from algebra, analysis and topology, and
- show, for example how the course generalizes results from ordinary differential equations.

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## Advanced Complex Function Theory (Q3+Q4)

### Objectives of the course

To continue with the classical theory of analytic functions in one complex variable, which was initiated in the course "Complex Function Theory".

### Course contents

This course is a continuation of the undergraduate course 'Complex Function Theory' and the contents is classical topics from the theory of function of one complex variable. A list of such topics are: Schwarz's Lemma and conformal mappings on the unit disc. The Phragmén-Lindelöf extension of the maximum principle. Spaces of analytic and meromorphic functions. The Riemann mapping Theorem. The Weierstrass Factorization Theorem (existence of analytic functions with prescribed zeroes). Runge's theorem. Mittag-Leffler's theorem (existence of meromorphic functions with prescribed poles). On the range of analytic functions there are Bloch's theorem and the little and great Picard theorems. Other possible subjects are Riemann's zeta-function and the prime number theorem, as well as harmonic functions and the Dirichlet problem.

### Prerequisites

Complex Function Theory and Real Analysis

### Name of lecturer

Søren Fournais

### Type of course / teaching methods

3 hours of lectures and 1 hour of exercises pr week

### Literature

M. Rao and H. Stetkær: *Complex analysis. An invitation.*, World Scientific, 1991.

## Language of instruction

English

## Course homepage

You will be able to find the course homepage at http://www.imf.au.dk before the course starts.

## Level of course

Graduate course

## Semester/quarter

3rd + 4th quarter (Spring 2012)

## Hours per week

4

## Capacity limits

## Assessment methods

The course will be evaluated using the Danish 7-scale with an internal examiner.

The evaluation is divided into two test points.

The first test point is either a written assignment that must be handed in to the teacher at the beginning of the second quarter or a written report on a given subject.

The second test point is an oral examination lasting about 20 minutes after 30 minutes preparation and with the use of all usual means of aid.

In determining the grade the first test points weighs 1/3, the oral exam weighs 2/3.

## Credits

10 ECTS

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher

## Provider

Department of Mathematical Sciences

## Course enrolment

At the self-service https://mit.au.dk

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- › reproduce key results and give rigorous and detailed proofs of them,
- › apply the basic techniques of complex function theory to concrete examples and exercises.

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## Functional equations II (Q3+4)

### Objectives of the course
To introduce the participants into recent developments in the theory of functional equations on topological groups.

### Course contents
The course is a continuation of the course "Functional equations" from the Fall of 2011. The previous course dealt mostly with functional equations on abelian groups. The new course shall discuss the situation on non-abelian groups.

### Prerequisites
Corresponding to the courses "Algebra" and "Measure theory" and the course "Functional equations" from the Fall of 2011.

### Name of lecturer
Henrik Stetkær

### Type of course / teaching methods
4 hours of lectures, exercises and seminars per week.

### Language of instruction
English

### Literature
Notes and articles from journals.

### Course homepage
You will be able to find the course homepage at http://www.imf.au.dk before the course starts.

### Level of course
Graduate course

### Semester/quarter

3rd + 4th quarter (Spring 2012)

## Hours per week

4

## Capacity limits

## Credits

10 ECTS

## Assessment methods

The course will be evaluated using the Danish 7-scale with an internal examiner.

The evaluation is divided into two test points. The first test point is a written assignment. The second test point is an oral examination lasting about 25 minutes after 30 minutes preparation with the use of all usual means of aid.

In determining the grade the first test point weighs 1/3, the oral exam weighs 2/3.

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher

## Provider

Department of Mathematical Sciences.

## Course enrolment

At the self-service https://mit.au.dk

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- › reproduce key results and give rigorous and detailed proofs of them,
- › compare key results,
- › apply the basic techniques, results and concepts of the course to concrete examples and exercises,
- › combine concepts from algebra, analysis and topology, and
- › show, how the course generalizes classical results.

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## Further Number Theory (Q3+Q4)

### Objectives of the course

To further develop the concepts and connections from the course Introduction to Number Theory, with a particular emphasis on modular forms, elliptic curves and Diophantine analysis.

### Course contents

The course builds on the concepts and ideas developed in the course Introduction to Number Theory. We focus on two fields, namely the interplay between analytic number theory and elliptic curves, and the classical study of Diophantine analysis.

Starting from the problem of congruent numbers, i.e., the possible integral areas of rectangular triangles with rational sides, elliptic curves and elliptic functions are introduces. These will be related to tools from analytic number theory via the Hasse-Weil $L$-function, which in turn is connected to modular forms via Hecke theory. We will also touch upon the famous Birch and Swinnerton-Dyer conjecture.

In Diophantine analysis, we will be concened with the interplay between Diophantine approximation and Diophantine equations. In its most basic form, Diophantine approximation concerns the approximation of real numbers by rational numbers. Diophantine equations on the other hand are equations in integers, where one looks for integer solutions. In this part of the course, we will cover the Fields medal winning results of Roth and Baker, and give applications of these to Diophantine equations as well as other branches of number theory.

### Prerequisites

The course Introduction to Number Theory

### Name of lecturer

Simon Kristensen and Alexei Venkov

### Type of course / teaching methods

4 hours of lectures per week.

## Literature

N. Koblitz: Introduction to elliptic curves and modular forms. Second edition. Graduate Texts in Mathematics, 97. Springer-Verlag, New York, 1993.
Supplementary notes.

## Language of instruction

English

## Course homepage

You will be able to find the course homepage at
http://www.imf.au.dk before the course starts.

## Level of course

Graduate course

## Semester/quarter

3rd + 4th quarter (Spring 2012)

## Hours per week

4

## Capacity limits

## Assessment methods

The course will be evaluated in two test points, both with an internal examiner. A single grade on the Danish 7-scale will be given.

The evaluation is divided into two test points.

The first test point is a larger written body of work (min. 8 pages), with a comprehensive treatement of a topic in number theory.

The second test point is an oral examination lasting about 20 minutes after 20 minutes preparation and with the use of all usual means of aid.

In determining the grade the first test points weighs 1/3, the oral exam weighs 2/3.

## Credits

10 ECTS

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher

## Provider

Department of Mathematical Sciences

## Course enrolment

At the self-service https://mit.au.dk

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- › reproduce key results in the field of number theory and give rigorous and detailed proofs of them,
- › compare key results within and across various branches of number theory,
- › apply the techniques, results and concepts of the course to concrete arithmetical examples and exercises,
- › combine concepts from analysis, algebra and arithmetic,
- › show how the course generalises the results from Introduction to Number Theory.

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## Project work in mathematics

### Objectives of the course

The purpose of the course is to enable the participants to make studies of special topics within mathematics. The topics are chosen in cooperation with a project supervisor, and the project is described by a short title. Through the work with the project the participants will be given an understanding of the techniques, results and concepts of the chosen topic.

### Compulsory programme

Made in agreement with the supervisor.

### Course contents

Varying from project to project.

### Prerequisites

Corresponding to the mathematical content of a bachelor degree in mathematics at Aarhus University.

### Name of lecturer

One or more of the teachers at the Department of Mathematical Sciences.

### Type of course / teaching methods

Several students can be on the project. It is taught by seminar lectures and exercises.

### Literature

Monographs, notes and papers.

## Course homepage

You will be able to find the course homepage at http://www.imf.au.dk/en before the course starts.

## Language of instruction

English

## Level of course

Graduate course

## Semester/quarter

All quarters

## Hours per week

Arranged in cooperation with the supervisor

## Capacity limits

None

## Assessment methods

At the examination the individual performance of the student is assessed using the 7-point grading scale.

The examination is an oral exam based on a written report. The time frame for the examination is 30 minutes and with the use of an internal examiner.

## Credits

5

ECTS

## Examination periods

The time for the examination is fixed in an agreement between the students and the supervisor.

## Provider

Department of Mathematical Sciences (IMF).

## Course enrolment

To sign up and get the form of the project fixed please contact Associate Professor Simon Kristensen, Department of Mathematical Sciences.

## Special comments on this course

At most one *Project work in mathematics* may be contained in the graduate studies.

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- › formulate problems within a limited subtopic
- › analyze the problems
- › work with the problems
- › communicate the results found to other persons
- › describe key mathematical results of the project and give rigorous, detailed proofs of them
- › apply the basic techniques, results and concepts to concrete examples and exercises.

GENERERET 07.11.2011 14:00:12

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8000 Århus C

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# WEBAROS:

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# UNDERVISNINGSBESKRIVELSE

## Cohomology and homotopy theory (Q3+Q4)

### Objectives of the course

To present fundamental methods of topology and homotopy theory.

### Compulsory programme

Participation in exercise sessions.

### Course contents

From an algebraic point of view, cohomology is a dual of homology. A cohomology class can be evaluated on a homology class to give a number. There are many situations where the, at first sight more complicated, cohomology turns out to be the more fundamental, simpler theory. There are a number of methods which properly belong to cohomology and to its interplay with homology. The most important one is the cup-product, which makes cohomology into a graded commutative ring.

We will also study homotopy theory, and in particular the relations between homotopy, homology, and cohomology groups. Important results here are Whitehead's theorem and Hurewicz' theorem.

### Prerequisites

The first two chapters of Hatcher's book, for instance from the course "Introduction to Algebraic Topology".

### Name of lecturer

Andrew du Plessis and Jørgen Tornehave

### Type of course / teaching methods

4 hours of lectures per week including exercises.

### Language of instruction

English

## Literature

Allen Hatcher: "Algebraic Topology", Cambridge University Press, 2002.

## Course homepage

You will be able to find the course homepage at https://www.imf.au.dk before the course starts.

## Level of course

Graduate course

## Semester/quarter

3rd + 4th quarter (Spring 2012)

## Hours per week

4

## Capacity limits

## Assessment methods

The course will be evaluated using the Danish 7-scale with an internal examiner.

The evaluation is divided into two test points.

The first test point is a written take home examination.

The second test point is an oral examination lasting about 20 minutes after 30 minutes preparation, with use of all the usual means of aid.

In determining the grade the first test points weighs 1/3, the oral exam weighs 2/3.

## Credits

10 ECTS

## Examination periods

Exam: 4th quarter

Re-exam: Contact the teacher

## Provider

Department of Mathematical Sciences.

## Course enrolment

At the self-service https://mit.au.dk

## Special comments on this course

None.

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- › reproduce key results and give rigorous and detailed proofs of them,
- › compare key results,
- › apply the basic techniques, results and concepts of the course to concrete examples and exercises,
- › combine concepts from algebra and topology.

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## Riemannian Geometry (Q3+Q4)

### Objectives of the course

The aim of the course is to cover the basics of Riemannian geometry with emphasis on reaching local-global results, for example the Gauss-Bonnet theorem or the Cartan-Hadamard theorem

### Compulsory programme

None.

### Course contents

The course will begin by reviewing the basics of general manifold theory, including tensors, vector bundles and differential forms. We will then proceed to cover the core of classical Riemannian geometry which includes metrics, connections, curvature and geodesics on Riemannian manifolds.

A primary goal of the course will be to prove certain local-global results that give relationships between the curvature and the topology of a manifold. Examples of such results include the Gauss-Bonnet Theorem for closed surfaces and the Theorem of Cartan and Hadamard for spaces of negative sectional curvature.

### Prerequisites

Geometry

### Name of lecturer

Andrew Swann and Jørgen Tornehave

### Type of course / teaching methods

3-4 lecture hours per week

## Literature

John M. Lee, "Riemannian Manifolds", An Introduction to Curvature, Springer Graduate Texts in Mathematics 176, New York, 1997.
(For further reading) Shigeyuki Morita, "Geometry of Differential Forms", Translations of the American Mathematical Society 201, Providence, RI, 2001.
Johan L. Dupont: "Differential Geometry", Lecture Notes no. 62, Aarhus University, 1993.

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## Course homepage

You will be able to find the course homepage at
http://www.imf.au.dk before the course starts.

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## Language of instruction

English

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## Level of course

Graduate course

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## Semester/quarter

3rd + 4th quarter (Spring 2012)

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## Hours per week

4

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## Capacity limits

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## Assessment methods

The course will be evaluated using the Danish grading scale (the 7-point scale) with an internal examiner.

The evaluation is divided into two test points.

The first test point is a written take-home exam.

The second test point is an oral examination lasting about 20 minutes after 30 minutes preparation and with the use of all usual means of aid.

In determining the grade the first test points weighs 1/3, the oral exam weighs 2/3.

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## Credits

10 ECTS

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## Examination periods

Exam: 4th quarter
Re-exam: Contact the teacher

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## Provider

Department of Mathematical Sciences

## Course enrolment

At the self-service https://mit.au.dk

## Learning outcomes and competences

Relevant to the course subject matter the student should at the end of the course be able to:

- › reproduce key results and give rigorous and detailed proofs of them,
- › apply the basic techniques, results and concepts of the course to concrete examples and exercises,
- › show, how the course generalizes classical results.

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